MA 261 **Exam 1** **Summer 2016**

Student's Name: KEY

Student's ID Number: ____________________

Instructions:

1. **Do NOT turn this page until told to do so.**
2. Show ALL work. No credit will be given without proper supporting work. For free response questions, write your final answer in the box provided. For multiple choice questions, clearly circle one answer.
3. There are 10 questions on 8 pages. Once you are allowed to turn the page, check that you have all pages.
4. No electronic devices, books, or notes are allowed. Please turn off your cell phone and put it away.
5. You will have 60 minutes to complete the exam.
6. **Keep your eyes on your own exam, and try to cover your work.**
7. Read the following statement and sign your name below.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately.

I have read and agree to these terms and conditions.

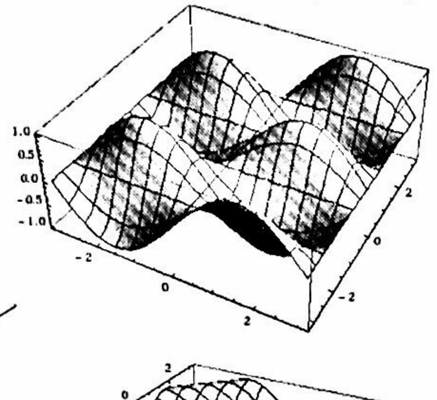
Signature ____________________

Grade	Cutoff
A	85
B	73
C	56
D	40
F	<40

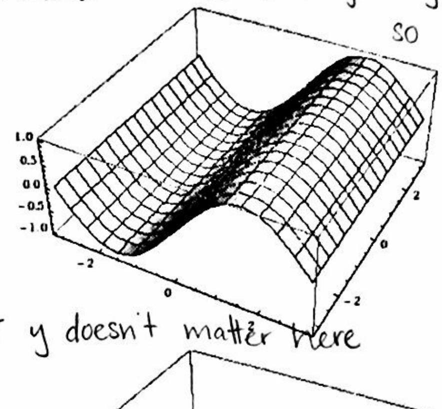
MA 261 - Exam 1

1. *(8 points)* Which of the following graphs is most like the graph of $f(x,y) = \sin(x-y)$? (x and y values vary from $-\pi$ to π.) Circle your answer clearly.

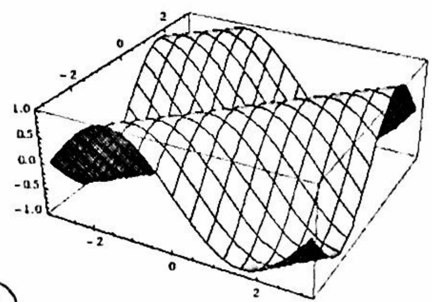
$k = \sin(x-y) \longrightarrow y = x - \sin^{-1} k$, so level curves are lines.



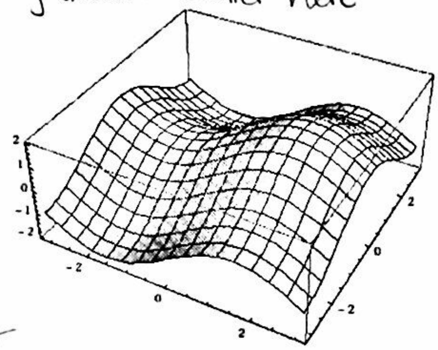
~~(a)~~



~~(c)~~ y doesn't matter here



(b)



~~(d)~~

2. *(8 points)* Find ~~$f_{xyz}(1,1)$~~ $f_{xyz}(1,1,3)$ if $f_{zx}(x,y) = x^2y - \ln(yz) + \sin^2\left(\frac{x^3}{z}\right)$, ~~and $f(1,1)=3$.~~

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

$f_{xyz} = (f_{zx})_y = x^2 - \frac{1}{yz} \cdot z$

$f_{xyz}(1,1,3) = 1^2 - \frac{1}{1} = -2$

3. *(8 points)* Let (a, b, c) be the point of intersection of the space curve $\vec{r}(t) = \langle t\sqrt{2}, t^2 + 1, 1 - 4t\rangle$ with the surface $x^2 + 2y - z = 0$. What is $2b + a^2$?

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

$$(t\sqrt{2})^2 + 2(t^2+1) - (1-4t) = 0$$
$$2t^2 + 2t^2 + 2 - 1 + 4t = 0$$
$$4t^2 + 4t + 1 = 0$$
$$t = \frac{-4 \pm \sqrt{16 - 4(4)(1)}}{2(4)} = -\frac{1}{2}$$
$$a = -\frac{1}{2}\sqrt{2}, \quad b = \left(-\frac{1}{2}\right)^2 + 1 = \frac{5}{4}$$
$$2b + a^2 = \frac{5}{2} + \frac{1}{2} = 3$$

4. *(8 points)* What is the name of the quadric surface given by $(x-2)^2 + (y-3)^2 - (z+4)^2 = 1$?

(a) ellipsoid

(b) hyperbolic paraboloid

(c) cone

(d) hyperboloid of one sheet

(e) hyperboloid of two sheets

one negative $\Rightarrow$ one sheet

5. *(12 points total)* The velocity of a particle moving through space is given by the vector function $\vec{v}(t) = \langle t^2, t^2e^{t^3}, -\sin \pi t\rangle$, and the initial position of the particle is $(0, 5, 0)$.

(a) *(7 points)* Find the position vector function, $\vec{r}(t)$

$$\vec{r}(t) = \int \vec{v}(t)dt = \langle \int t^2 dt, \int t^2e^{t^3}dt, \int -\sin \pi t \, dt \rangle$$
$$= \langle \tfrac{1}{3}t^3 + C_1, \tfrac{1}{3}e^{t^3} + C_2, \tfrac{1}{\pi}\cos \pi t + C_3 \rangle \quad 4$$

$$\vec{r}(0) = \langle C_1, \tfrac{1}{3} + C_2, \tfrac{1}{\pi} + C_3 \rangle = \langle 0, 5, 0 \rangle$$

$$C_1 = 0$$
$$C_2 = \frac{14}{3}$$
$$C_3 = -\frac{1}{\pi}$$

$$\boxed{\vec{r}(t) = \langle \tfrac{1}{3}t^3, \tfrac{1}{3}e^{t^3} + \tfrac{14}{3}, \tfrac{1}{\pi}\cos \pi t - \tfrac{1}{\pi} \rangle}$$

(b) *(5 points)* Find the acceleration vector function, $\vec{a}(t)$

$$\vec{a}(t) = \vec{v}'(t) = \langle 2t, 2te^{t^3} + 3t^2(t^2e^{t^3}), -\pi \cos \pi t \rangle$$

$$\boxed{\vec{a}(t) = \langle 2t, e^{t^3}(2t + 3t^4), -\pi \cos \pi t \rangle}$$

6. *(10 points)* Use the linear approximation of $f(x, y) = x^2 e^y$ at the point $(1, 0)$ to approximate $\frac{0.81}{e^{0.1}}$. (Hint: $f(0.9, -0.1) = \frac{0.81}{e^{0.1}}$).

$f_x = 2xe^y$ $f_y = x^2 e^y$ $f(1,0) = 1$

$f_x(1,0) = 2$ $f_y(1,0) = 1$

$$f(x,y) \approx f(1,0) + f_x(1,0)(x-1) + f_y(1,0)(y-0)$$
$$= 1 + 2(x-1) + y$$
$$f(0.9,-.1) \approx 1 + 2(-.1) + (-.1) = .7$$

$$\boxed{f(0.9,-.1) \approx .7}$$

7. *(10 points)* Use Chain Rule to find $\frac{\partial w}{\partial r}$ if $w = xy + yz + zx$, $x = r\cos\theta$, $y = r\sin\theta$, and $z = \theta$.

$$\frac{dw}{dr} = \frac{dw}{dx}\frac{dx}{dr} + \frac{dw}{dy}\frac{dy}{dr} + \frac{dw}{dz}\frac{dz}{dr}$$
$$= (y+z)(\cos\theta) + (x+z)\sin\theta + (y+x)(0)$$

$$\boxed{\frac{dw}{dr} = x\sin\theta + y\cos\theta + z(\sin\theta + \cos\theta)}$$

8. *(14 points total)*

(a) *(7 points)* If $\vec{r}(t) = \langle 3t, 4\sin t, 4\cos t\rangle$, then $\vec{T}(t) = \langle \frac{3}{5}, \frac{4}{5}\cos t, -\frac{4}{5}\sin t\rangle$. Find $\vec{N}(t)$.

$$\vec{T}'(t) = \langle 0, -\tfrac{4}{5}\sin t, -\tfrac{4}{5}\cos t\rangle$$

$$|\vec{T}'(t)| = \sqrt{(-\tfrac{4}{5}\sin t)^2 + (-\tfrac{4}{5}\cos t)^2} = \frac{4}{5}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\langle 0, -\frac{4}{5}\sin t, -\frac{4}{5}\cos t\rangle}{4/5}$$

$$\boxed{\vec{N}(t) = \langle 0, -\sin t, -\cos t\rangle}$$

(b) *(7 points)* If $\vec{r}(t) = \langle e^t, t\sqrt{2}, e^{-t}\rangle$, then $\vec{T}'(t) = \frac{1}{(e^t + e^{-t})^2}\langle 2, \sqrt{2}(e^t - e^{-t}), -2\rangle$. Find the curvature of $\vec{r}(t)$. $\left(\text{Recall } \kappa = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}.\right)$

$$|\vec{T}'(t)| = \frac{1}{(e^t+e^{-t})^2}\sqrt{2^2 + (\sqrt{2}(e^t - e^{-t}))^2 + (-2)^2}$$

$$= \frac{1}{(e^t+e^{-t})^2}\sqrt{8 + 2(e^{2t} - 2 + e^{-2t})}$$

$$= \frac{1}{(e^t+e^{-t})^2}\sqrt{2(e^t+e^{-t})^2}$$

$$= \frac{\sqrt{2}}{e^t+e^{-t}}$$

$$\vec{r}'(t) = \langle e^t, \sqrt{2}, -e^{-t}\rangle$$

$$|\vec{r}'(t)| = \sqrt{e^{2t} + 2 + e^{-2t}}$$

$$= \sqrt{(e^t+e^{-t})^2}$$

$$= e^t + e^{-t}$$

$$\kappa = \frac{\sqrt{2}/e^t+e^{-t}}{e^t+e^{-t}}$$

$$\boxed{\kappa = \frac{\sqrt{2}}{(e^t+e^{-t})^2}}$$

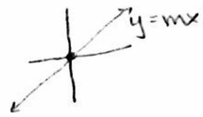
9. *(12 points total)* Compute the following limits. If a limit does not exist, write DNE for your answer and show in your work at least two different values that it can attain.

(a) *(4 points)* $\lim\limits_{(x,y)\to(0,0)} \dfrac{xe^y}{\sin x + 1} = \dfrac{0}{1} = 0$

because $\dfrac{xe^y}{\sin x+1}$ is continuous at $(0,0)$.

$\boxed{0}$

(b) *(4 points)* $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{x^2-y^2} = \dfrac{0}{0}$

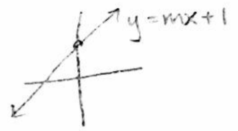


along $x=0$: $\lim\limits_{y\to 0} \dfrac{y^2}{-y^2} = -1$

along $y=mx$: $\lim\limits_{x\to 0} \dfrac{x^2+m^2x^2}{x^2-m^2x^2} = \lim\limits_{x\to 0} \dfrac{1+m^2}{1-m^2} = 1$ if $m=0$

$\boxed{\text{DNE}}$

(c) *(4 points)* $\lim\limits_{(x,y)\to(0,1)} \dfrac{xy-x}{x^2+(y-1)^2} = \dfrac{0}{0}$



along $x=0$: $\lim\limits_{y\to 1} \dfrac{0}{(y-1)^2} = 0$

along $y=mx+1$: $\lim\limits_{x\to 0} \dfrac{x(mx+1)-x}{x^2+(mx+1-1)^2} = \lim\limits_{x\to 0} \dfrac{mx^2}{(1+m^2)x^2} = \dfrac{m}{1+m^2} = \dfrac{1}{2}$ if $m=1$

$\boxed{\text{DNE}}$

10. *(10 points)* Find parametric equations for the line of intersection of the planes $x+3y-z=1$ and $-x-y+2z=17$.

the line of intersection is ortho to both normal vectors:

$\langle 1,3,-1\rangle \times \langle -1,-1,2\rangle = \begin{array}{cccccc} 1 & 3 & -1 & 1 & 3 & -1 \\ -1 & -1 & 2 & -1 & -1 & 2 \end{array} = \langle 5,-1,2\rangle$

we need a point of intersection:

Let $x=0$

$$\begin{cases} 3y-z=1 \longrightarrow z = 3y+1 \\ -y+2z=17 \longrightarrow -y+2(3y+1)=17 \end{cases}$$

$5y=15$

$y=3$ $\quad (0,3,10)$

$z=10$

$\vec{r}(t) = \langle 0+5t, 3+(-1)t, 10+2t\rangle$

$$\boxed{x=5t,\ y=3-t,\ z=10+2t}$$

Alternative Method:

Let $z=t$.

$$\begin{cases} x+3y-t=1 & ① \\ -x-y+2t=17 & ② \end{cases}$$

①+② $\quad 2y+t=18$, so $y=9-\frac{1}{2}t$

①+3② $\quad -2x+5t=52$, so $x=-26+\frac{5}{2}t$

$$\boxed{x=-26+\frac{5}{2}t,\ y=9-\frac{1}{2}t,\ z=t}$$